PMT

Mark Scheme 4753
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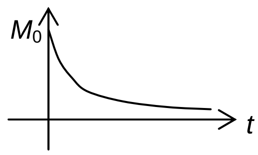
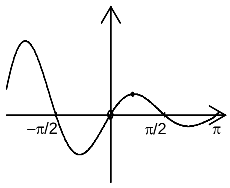
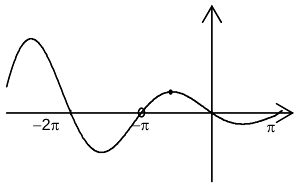
1 $\lvert 3x-2\rvert = x$ $\Rightarrow$ $3x-2=x \Rightarrow 2x=2 \Rightarrow x=1$ or $2-3x=x \Rightarrow 2=4x \Rightarrow x=½$ *or* $(3x-2)^2=x^2$ $\Rightarrow 8x^2-12x+4=0 \Rightarrow 2x^2-3x+1=0$ $\Rightarrow (x-1)(2x-1)=0,$ $\Rightarrow x = 1, ½$	B1 M1 A1 M1 A1 A1 [3]	$x = 1$ solving correct quadratic
2 let $u = x$, $dv/dx = \sin 2x \Rightarrow v = -½\cos 2x$ $\Rightarrow \int_0^{\pi/6} x\sin 2x dx = \left[x.-\frac{1}{2}\cos 2x\right]_0^{\pi/6} + \int_0^{\pi/6}\frac{1}{2}\cos 2x.1.dx$ $= \frac{\pi}{6}.-\frac{1}{2}\cos\frac{\pi}{3} - 0 + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$ $= -\frac{\pi}{24} + \frac{\sqrt{3}}{8}$ $= \frac{3\sqrt{3}-\pi}{24}$ *	M1 A1 B1ft M1 B1 E1 [6]	parts with $u = x$, $dv/dx = \sin 2x$ $...+\left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$ substituting limits $\cos \pi/3 = ½$, $\sin \pi/3 = \sqrt{3}/2$ soi www
3 (i) $x - 1 = \sin y$ $\Rightarrow$ $x = 1 + \sin y$ $\Rightarrow$ $dx/dy = \cos y$ **(ii)** When $x = 1.5$, $y = \arcsin(0.5) = \pi/6$ $\frac{dy}{dx} = \frac{1}{\cos y}$ $= \frac{1}{\cos \pi/6}$ $= 2/\sqrt{3}$	M1 A1 E1 M1 A1 M1 A1 [7]	 www condone 30° or 0.52 or better or $\frac{dy}{dx} = \frac{1}{\sqrt{1-(x-1)^2}}$ or equivalent, but must be exact
4(i) $V = \pi h^2 - \frac{1}{3}\pi h^3$ $\Rightarrow \frac{dV}{dh} = 2\pi h - \pi h^2$ **(ii)** $\frac{dV}{dt} = 0.02$ $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$ $\Rightarrow \frac{dh}{dt} = \frac{0.02}{dV/dh} = \frac{0.02}{2\pi h - \pi h^2}$ When $h = 0.4$, $\Rightarrow \frac{dh}{dt} = \frac{0.02}{0.8\pi - 0.16\pi} = 0.0099\,\text{m/min}$	M1 A1 B1 M1 M1dep A1cao [6]	expanding brackets (correctly) or product rule oe soi $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$ oe substituting $h = 0.4$ into their $\frac{dV}{dh}$ and $\frac{dV}{dt} = 0.02$ 0.01 or better or $1/32\pi$

5(i) $a^2 + b^2 = (2t)^2 + (t^2 - 1)^2$ $= 4t^2 + t^4 - 2t^2 + 1$ $= t^4 + 2t^2 + 1$ $= (t^2 + 1)^2 = c^2$	M1 M1 E1	substituting for a, b and c in terms of t Expanding brackets correctly www
(ii) $c = \sqrt{(20^2 + 21^2)} = 29$ For example: $2t = 20 \Rightarrow t = 10$ $\Rightarrow$ $t^2 - 1 = 99$ which is not consistent with 21	B1 M1 E1 [6]	 Attempt to find t Any valid argument or E2 'none of 20, 21, 29 differ by two'.
6 (i) 	B1 B1	Correct shape Passes through $(0, M_0)$
(ii) $\frac{M}{M_0} = e^{-0.000121\times5730} = e^{-0.6933...} \approx \frac{1}{2}$	M1 E1	substituting $k = -0.00121$ and $t = 5730$ into equation (or ln eqn) showing that $M \approx \frac{1}{2} M_0$
(iii) $\frac{M}{M_0} = e^{-kT} = \frac{1}{2}$ $\Rightarrow \ln\frac{1}{2} = -kT$ $\Rightarrow \ln 2 = kT$ $\Rightarrow T = \frac{\ln 2}{k}$ *	M1 M1 E1	substituting $M/M_0 = \frac{1}{2}$ into equation (oe) taking lns correctly
(iv) $T = \frac{\ln 2}{2.88\times10^{-5}} \approx 24\,000$ years	B1 [8]	24 000 or better

Section B

7(i) $x = 1$	B1 [1]	
(ii) $\frac{dy}{dx} = \frac{(x-1)2x-(x^2+3).1}{(x-1)^2}$	M1	Quotient rule
$= \frac{2x^2-2x-x^2-3}{(x-1)^2}$	A1	correct expression
$= \frac{x^2-2x-3}{(x-1)^2}$		
$dy/dx = 0$ when $x^2 - 2x - 3 = 0$	M1	their numerator = 0
$\Rightarrow (x-3)(x+1) = 0$	M1	solving quadratic by any valid method
$\Rightarrow x = 3$ or -1	A1	$x = 3$ from correct working
When $x = 3$, $y = (9 + 3)/2 = 6$ So P is (3, 6)	B1ft [6]	$y = 6$
(iii) Area $= \int_2^3 \frac{x^2+3}{x-1}dx$	M1	Correct integral and limits
$u = x - 1 \Rightarrow du/dx = 1$, $du = dx$ When $x = 2$, $u = 1$; when $x = 3$, $u = 2$ $= \int_1^2 \frac{(u+1)^2+3}{u}du$	B1	Limits changed, and substituting $dx = du$
$= \int_1^2 \frac{u^2+2u+4}{u}du$	B1	substituting $\frac{(u+1)^2+3}{u}$
$= \int_1^2 (u+2+\frac{4}{u})du$ *	E1	www
$= \left[\frac{1}{2}u^2+2u+4\ln u\right]_1^2$	B1	[$\frac{1}{2}u^2 + 2u + 4\ln u$]
$= (2 + 4 + 4\ln 2) - (\frac{1}{2} + 2 + 4\ln 1)$	M1	substituting correct limits
$= 3\frac{1}{2} + 4\ln 2$	A1cao [7]	
(iv) $e^y = \frac{x^2+3}{x-1}$		
$\Rightarrow e^y\frac{dy}{dx} = \frac{x^2-2x-3}{(x-1)^2}$	M1	$e^y dy/dx$ = their $f'(x)$ or $xe^y - e^y = x^2 + 3$
$\Rightarrow \frac{dy}{dx} = e^{-y}\frac{x^2-2x-3}{(x-1)^2}$	A1ft	$\Rightarrow e^y + xe^y\frac{dy}{dx} - e^y\frac{dy}{dx} = 2x$ $\Rightarrow \frac{dy}{dx} = \frac{2x-e^y}{e^y(x-1)}$
When $x = 2$, $e^y = 7 \Rightarrow$	B1	$y = \ln 7$ or 1.95… or $e^y = 7$
$\Rightarrow dy/dx = \frac{1}{7}.\frac{4-4-3}{1} = -\frac{3}{7}$	A1cao [4]	or $\frac{dy}{dx} = \frac{4-7}{7(2-1)} = -\frac{3}{7}$ or -0.43 or better

8 (i) (*A*)  (*B*) 	B1 B1 M1 A1 [4]	Zeros shown every π/2. Correct shape, from −π to π Translated in *x*-direction π to the left
(ii) $f'(x) = -\frac{1}{5}e^{-\frac{1}{5}x}\sin x + e^{-\frac{1}{5}x}\cos x$ $f'(x) = 0$ when $-\frac{1}{5}e^{-\frac{1}{5}x}\sin x + e^{-\frac{1}{5}x}\cos x = 0$ $\Rightarrow \frac{1}{5}e^{-\frac{1}{5}x}(-\sin x + 5\cos x) = 0$ $\Rightarrow \sin x = 5\cos x$ $\Rightarrow \frac{\sin x}{\cos x} = 5$ $\Rightarrow \tan x = 5$* $\Rightarrow x = 1.37(34\ldots)$ $\Rightarrow y = 0.75$ or $0.74(5\ldots)$	B1 B1 M1 E1 B1 B1 [6]	$e^{-\frac{1}{5}x}\cos x$ $\ldots -\frac{1}{5}e^{-\frac{1}{5}x}\sin x$ dividing by $e^{-\frac{1}{5}x}$ www 1.4 or better, must be in radians 0.75 or better
(iii) $f(x+\pi) = e^{-\frac{1}{5}(x+\pi)}\sin(x+\pi)$ $= e^{-\frac{1}{5}x}e^{-\frac{1}{5}\pi}\sin(x+\pi)$ $= -e^{-\frac{1}{5}x}e^{-\frac{1}{5}\pi}\sin x$ $= -e^{-\frac{1}{5}\pi}f(x)$ * $\int_{\pi}^{2\pi} f(x)dx$ let $u = x - \pi$, $du = dx$ $= \int_0^{\pi} f(u+\pi)du$ $= \int_0^{\pi} -e^{-\frac{1}{5}\pi}f(u)du$ $= -e^{-\frac{1}{5}\pi}\int_0^{\pi} f(u)du$ * Area enclosed between π and 2π $= (-)e^{-\frac{1}{5}\pi} \times$ area between 0 and π.	M1 A1 A1 E1 B1 B1dep E1 B1 [8]	$e^{-\frac{1}{5}(x+\pi)} = e^{-\frac{1}{5}x}.e^{-\frac{1}{5}\pi}$ $\sin(x+\pi) = -\sin x$ www $\int f(u+\pi)du$ limits changed using above result or repeating work or multiplied by 0.53 or better